5.3 ASSESS YOUR UNDERSTANDING

Concepts and Vocabulary

1. Two events E and F are __________ if the occurrence of event E in a probability experiment does not affect the probability of event F.
2. The word *and* in probability implies that we use the __________ Rule.
3. The word *or* in probability implies that we use the ________ Rule.
4. *True or False*: When two events are disjoint, they are also independent.
5. If two events E and F are independent, $P(E \text{ and } F) =$ ________.
6. Suppose events E and F are disjoint. What is $P(E \text{ and } F)$?

Skill Building

7. Determine whether the events E and F are independent or NW dependent. Justify your answer.
   (a) E: Speeding on the interstate.
   F: Being pulled over by a police officer.
   (b) E: You gain weight.
   F: You eat fast food for dinner every night.
   (c) E: You get a high score on a statistics exam.
   F: The Boston Red Sox win a baseball game.
8. Determine whether the events E and F are independent or dependent. Justify you answer.
   (a) E: The battery in your cell phone is dead.
   F: The batteries in your calculator are dead.
   (b) E: Your favorite color is blue.
   F: Your friend's favorite hobby is fishing.
   (c) E: You are late for school.
   F: Your car runs out of gas.
9. Suppose that events E and F are independent, $P(E) = 0.3$, and $P(F) = 0.6$. What is the $P(E \text{ and } F)$?
10. Suppose that events E and F are independent, $P(E) = 0.7$, and $P(F) = 0.9$. What is the $P(E \text{ and } F)$?

Applying the Concepts

11. **Flipping a Coin** What is the probability of obtaining five heads in a row when flipping a coin? Interpret this probability.
12. **Rolling a Die** What is the probability of obtaining 4 ones in a row when rolling a fair, six-sided die? Interpret this probability.
13. **Southpaws** About 13% of the population is left-handed. If two people are randomly selected, what is the probability that both are left-handed? What is the probability that at least one is right-handed?
14. **Double Jackpot** Shawn lives near the border of Illinois and Missouri. One weekend he decides to play \$1 in both state lotteries in hopes of hitting two jackpots. The probability of winning the Missouri Lotto is about 0.00000028357 and the probability of winning the Illinois Lotto is about 0.000000098239.
    (a) Explain why the two lotteries are independent.
    (b) Find the probability that Shawn will win both jackpots.
15. **False Positives** The ELISA is a test to determine whether the HIV antibody is present. The test is 99.5% effective. This means that the test will accurately come back negative if the HIV antibody is not present. The probability of a test coming back positive when the antibody is not present (a false positive) is 0.005. Suppose that the ELISA is given to five randomly selected people who do not have the HIV antibody.
    (a) What is the probability that the ELISA comes back negative for all five people?
    (b) What is the probability that the ELISA comes back positive for at least one of the five people?
16. **Christmas Lights** Christmas lights are often designed with a series circuit. This means that when one light burns out the entire string of lights goes black. Suppose that the lights are designed so that the probability a bulb will last 2 years is 0.995. The success or failure of a bulb is independent of the success or failure of other bulbs.
    (a) What is the probability that in a string of 100 lights all 100 will last 2 years?
    (b) What is the probability that at least one bulb will burn out in 2 years?
17. **Life Expectancy** The probability that a randomly selected NW 40-year-old male will live to be 41 years old is 0.99757, according to the *National Vital Statistics Report*, Vol. 56, No. 9.
    (a) What is the probability that two randomly selected 40-year-old males will live to be 41 years old?
    (b) What is the probability that five randomly selected 40-year-old males will live to be 41 years old?
    (c) What is the probability that at least one of five randomly selected 40-year-old males will not live to be 41 years old? Would it be unusual if at least one of five randomly selected 40-year-old males did not live to be 41 years old?
18. **Life Expectancy** The probability that a randomly selected 40-year-old female will live to be 41 years old is 0.99855 according to the *National Vital Statistics Report*, Vol. 56, No. 9.
    (a) What is the probability that two randomly selected 40-year-old females will live to be 41 years old?
    (b) What is the probability that five randomly selected 40-year-old females will live to be 41 years old?
    (c) What is the probability that at least one of five randomly selected 40-year-old females will not live to be 41 years old? Would it be unusual if at least one of five randomly selected 40-year-old females did not live to be 41 years old?
19. **Blood Types** Blood types can be classified as either Rh^+ or Rh^-. According to the *Information Please Almanac*, 99% of the Chinese population has Rh^+ blood.
    (a) What is the probability that two randomly selected Chinese people have Rh^+ blood?
    (b) What is the probability that six randomly selected Chinese people have Rh^+ blood?
    (c) What is the probability that at least one of six randomly selected Chinese people has Rh^- blood? Would it be unusual that at least one of six randomly selected Chinese people has Rh^- blood?
20. **Quality Control** Suppose that a company selects two people who work independently inspecting two-by-four timbers.

Their job is to identify low-quality timbers. Suppose that the probability that an inspector does not identify a low-quality timber is 0.20.

(a) What is the probability that both inspectors do not identify a low-quality timber?
(b) How many inspectors should be hired to keep the probability of not identifying a low-quality timber below 1%?
(c) Interpret the probability from part (a).

21. Reliability For a parallel structure of identical components, the system can succeed if at least one of the components succeeds. Assume that components fail independently of each other and that each component has a 0.15 probability of failure.

(a) Would it be unusual to observe one component fail? Two components?
(b) What is the probability that a parallel structure with 2 identical components will succeed?
(c) How many components would be needed in the structure so that the probability the system will succeed is greater than 0.9999?

22. E.P.T. Pregnancy Tests The packaging of an E.P.T. Pregnancy Test states that the test is "99% accurate at detecting typical pregnancy hormone levels." Assume that the probability that a test will correctly identify a pregnancy is 0.99 and that 12 randomly selected pregnant women with typical hormone levels are each given the test.

(a) What is the probability that all 12 tests will be positive?
(b) What is the probability that at least one test will not be positive?

23. Cold Streaks Players in sports are said to have "hot streaks" and "cold streaks." For example, a batter in baseball might be considered to be in a slump, or cold streak, if he has made 10 outs in 10 consecutive at-bats. Suppose that a hitter successfully reaches base 30% of the time he comes to the plate.

(a) Find the probability that the hitter makes 10 outs in 10 consecutive at-bats, assuming that at-bats are independent events. *Hint*: The hitter makes an out 70% of the time.
(b) Are cold streaks unusual?
(c) Interpret the probability from part (a).

24. Hot Streaks In a recent basketball game, a player who makes 65% of his free throws made eight consecutive free throws. Assuming that free-throw shots are independent, determine whether this feat was unusual.

25. Bowling Suppose that Ralph gets a strike when bowling 30% of the time.

(a) What is the probability that Ralph gets two strikes in a row?
(b) What is the probability that Ralph gets a turkey (three strikes in a row)?
(c) When events are independent, their complements are independent as well. Use this result to determine the probability that Ralph gets a strike and then does not get a strike.

26. NASCAR Fans Among Americans who consider themselves auto racing fans, 59% identify NASCAR stock cars as their favorite type of racing. Suppose that four auto racing fans are randomly selected.
Source: ESPN/TNS Sports, reported in *USA Today*

(a) What is the probability that all four will identify NASCAR stock cars as their favorite type of racing?
(b) What is the probability that at least one will not identify NASCAR stock cars as his or her favorite type of racing?
(c) What is the probability that none will identify NASCAR stock cars as his or her favorite type of racing?
(d) What is the probability that at least one will identify NASCAR stock cars as his or her favorite type of racing?

27. Driving under the Influence Among 21- to 25-year-olds, 29% say they have driven while under the influence of alcohol. Suppose that three 21- to 25-year-olds are selected at random.
Source: U.S. Department of Health and Human Services, reported in *USA Today*

(a) What is the probability that all three have driven while under the influence of alcohol?
(b) What is the probability that at least one has not driven while under the influence of alcohol?
(c) What is the probability that none of the three has driven while under the influence of alcohol?
(d) What is the probability that at least one has driven while under the influence of alcohol?

28. Defense System Suppose that a satellite defense system is established in which four satellites acting independently have a 0.9 probability of detecting an incoming ballistic missile. What is the probability that at least one of the four satellites detects an incoming ballistic missile? Would you feel safe with such a system?

29. Audits For the fiscal year 2007, the IRS audited 1.77% of individual tax returns with income of $100,000 or more. Suppose this percentage stays the same for the current tax year.

(a) Would it be unusual for a return with income of $100,000 or more to be audited?
(b) What is the probability that two randomly selected returns with income of $100,000 or more will be audited?
(c) What is the probability that two randomly selected returns with income of $100,000 or more will *not* be audited?
(d) What is the probability that at least one of two randomly selected returns with income of $100,000 or more will be audited?

30. Casino Visits According to a December 2007 Gallup poll, 24% of American adults have visited a casino in the past 12 months.

(a) What is the probability that 4 randomly selected adult Americans have visited a casino in the past 12 months? Is this result unusual?
(b) What is the probability that 4 randomly selected adult Americans have *not* visited a casino in the past 12 months? Is this result unusual?

31. Betting on Sports According to a Gallup Poll, about 17% of adult Americans bet on professional sports. Census data indicate that 48.4% of the adult population in the United States is male.

(a) Are the events "male" and "bet on professional sports" mutually exclusive? Explain.
(b) Assuming that betting is independent of gender, compute the probability that an American adult selected at random is a male and bets on professional sports.
(c) Using the result in part (b), compute the probability that an American adult selected at random is male or bets on professional sports.

(d) The Gallup poll data indicated that 10.6% of adults in the United States are males and bet on professional sports. What does this indicate about the assumption in part (b)?

(e) How will the information in part (d) affect the probability you computed in part (c)?

32. Fingerprints Fingerprints are now widely accepted as a form of identification. In fact, many computers today use fingerprint identification to link the owner to the computer. In 1892, Sir Francis Galton explored the use of fingerprints to uniquely identify an individual. A fingerprint consists of ridgelines. Based on empirical evidence, Galton estimated the probability that a square consisting of six ridgelines that covered a fingerprint could be filled in accurately by an experienced fingerprint analyst as $\frac{1}{2}$.

(a) Assuming that a full fingerprint consists of 24 of these squares, what is the probability that all 24 squares could be filled in correctly; assuming that success or failure in filling in one square is independent of success or failure in filling in any other square within the region? (This value represents the probability that two individuals would share the same ridgeline features within the 24-square region.)

(b) Galton further estimated that the likelihood of determining the fingerprint type (e.g. arch, left loop, whorl, etc.) as $\left(\frac{1}{2}\right)^4$ and the likelihood of the occurrence of the correct number of ridges entering and exiting each of the 24 regions as $\left(\frac{1}{2}\right)^8$. Assuming that all three probabilities are independent, compute Galton's estimate of the probability that a particular fingerprint configuration would occur in nature (that is, the probability that a fingerprint match occurs by chance).

5.4 CONDITIONAL PROBABILITY AND THE GENERAL MULTIPLICATION RULE

Objectives

1. Compute conditional probabilities
2. Compute probabilities using the General Multiplication Rule

1 Compute Conditional Probabilities

Note to Instructor
For Instructors looking to offer a course that minimizes the coverage of probability, this section can be skipped without loss of continuity.

In the last section, we learned that when two events are independent the occurrence of one event has no effect on the probability of the second event. However, we cannot generally assume that two events will be independent. Will the probability of being in a car accident change depending on driving conditions? We would expect so. For example, we would expect the probability of an accident to be higher for nighttime driving on icy roads than of daytime driving on dry roads. What about the likelihood of contracting a sexually transmitted disease (STD)? Do you think the number of sexual partners will affect the likelihood of contracting an STD?

According to data from the Centers for Disease Control, 33.3% of adult men in the United States are obese. So the probability is 0.333 that a randomly selected adult male in the United States is obese. However, 28% of adult men aged 20 to 39 are obese compared to 40% of adult men aged 40 to 59. The probability is 0.28 that an adult male is obese, *given* that they are aged 20 to 39. The probability is 0.40 that an adult male is obese, *given* that they are aged 40 to 59. The probability that an adult male is obese changes depending on the age group in which the individual falls. This is called *conditional probability*.

Definition

Conditional Probability

The notation $P(F|E)$ is read "the probability of event F given event E." It is the probability that the event F occurs, given that the event E has occurred.

Let's look at an example.